Orange-Ulster BOCES
and
SUNY Orange
Faculty Exchange
Thursday, November 29, 2007
Prepared by
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Math Chair Middletown High School
Mathematics, Science, and Technology

Learning Standard 3: Mathematics

Students will:

• understand the concepts of and become proficient with the skills of mathematics;
• communicate and reason mathematically;
• become problem solvers by using appropriate tools and strategies; through the integrated study of number sense and operations, algebra, geometry, measurement, and statistics and probability.
Standard 3

The Three Components

• **Conceptual Understanding** consists of those relationships constructed internally and connected to already existing ideas.

• **Procedural Fluency** is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

• **Problem Solving** is the ability to formulate, represent, and solve mathematical problems.
### Standard 3

**Process and Content Strands**

<table>
<thead>
<tr>
<th>The Five Process Strands</th>
<th>The Five Content Strands</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving</td>
<td>Number Sense and Operations</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td>Algebra</td>
</tr>
<tr>
<td>Communication</td>
<td>Geometry</td>
</tr>
<tr>
<td>Connections</td>
<td>Measurement</td>
</tr>
<tr>
<td>Representation</td>
<td>Statistics and Probability</td>
</tr>
</tbody>
</table>
New York State Mathematics Standard 3

Content Strands

Process Strands
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

Mathematical Proficiency
- Conceptual Understanding
- Procedural Fluency
- Problem Solving

Math Credits

- **Old SED Curriculum**
  - Math A - 1.5 to 3 credits
  - Math B - 1.5 to 2 credits
  - Above - up to individual school district

- **New SED Curriculum**
  - Integrated Algebra 1 to 2 credits
  - Integrated Geometry 1 to 2 credits
  - Integrated Algebra 2 and Trigonometry
  - Above - up to individual school district
Graduation Requirement

**Local Diploma**
- 3 credits of math
- 1 regents exam in mathematics
  - 55 to 64 on any math regents exam
  - RCT in math

**Regents Diploma**
- 3 credits of math
- 1 regents exam in mathematics 65 or higher

**Regents Diploma with Advanced Designation**
- 3 credits of math
- 3 regents exams in math 65 or higher
NYS High School Faculty

- State Mandated Curriculum
- Local Scope and Sequences
- State Mandated Exams
- Local Final Exams
- State Mandated Graphing Calculator
March 2007

TO: Superintendents and Principals
FROM: Jean C. Stevens
SUBJECT: The Use of Calculators in Commencement Level Mathematics

In March 2005, the Board of Regents adopted a new Learning Standard for Mathematics …

As schools prepare for the changes in mathematics, a continuing theme has been the use of educational technologies for improved student achievement and as a powerful student motivator. Calculators are instrumental as an investigative tool in the teaching and learning of mathematics to enhance students’ conceptual understanding.
The graphing calculator should be used for all types of classroom activities and homework as an integral part of the three Regents-approved commencement level mathematics courses, Integrated Algebra, Geometry, and Algebra 2/Trigonometry.

A graphing calculator must be made available for use by each student while taking the corresponding Regents Examination.
Integrated Algebra

Curriculum
*Integrated Algebra* is the first mathematics course in the high school. This integrated algebra course is **not** the algebra of 30 years ago.

Algebra provides tools and ways of thinking that are necessary for solving problems in a wide variety of disciplines, such as science, business, social sciences, fine arts, and technology.

This course will assist students in developing skills and processes to be applied using a variety of techniques to successfully solve problems in a variety of settings.
Alternative solution methods should be given equal value within the strategies used for problem solving.

Problem situations may result in all types of linear equations in one variable, quadratic functions with integral coefficients and roots as well as absolute value and exponential functions.

Problem situations resulting in systems of equations will also be presented.

Coordinate geometry will be integrated allowing students to make connections between their analytical and geometrical representations.
Data analysis including measures of central tendency and visual representations of data will be studied. An understanding of correlation and causation will be developed and reasonable lines of best fit will be used to make predictions.

Students will solve problem situations requiring right triangle trigonometry.

Elementary probability theory will be used to determine the probability of events including independent, dependent and mutually exclusive events.
Integrated Algebra

- Real Numbers
- Solutions of Linear Equations & Inequalities
- Word Problems
- Graphing Two Variable Linear Equations and Inequalities
- Equations of Lines
- Systems of Linear Equations or Inequalities
- Polynomials
- Rational Expressions
Integrated Algebra continued

- Radicals
- Parabolas
- Systems of Quadratic–Linear Equations
- Right Triangles
- Perimeter, Area, Volume, and Total Surface Area
- Probability
- Sets & Functions
- Statistics
Integrated Geometry
Geometry is intended to be the second course in mathematics for high school students. There is no other school math course that offers students the opportunity to act as mathematicians.

Within this course, students will have the opportunity to make conjectures about geometric situations and prove in a variety of ways, both formal and informal, that their conclusion follows logically from their hypothesis.

This course is meant to employ an integrated approach to the study of geometric relationships. Integrating synthetic, transformational, and coordinate approaches to geometry, students will justify geometric relationships and properties of geometric figures. Congruence and similarity of triangles will be established using appropriate theorems.
Transformations including rotations, reflections, translations, and glide reflections and coordinate geometry will be used to establish and verify geometric relationships.

A major emphasis of this course is to allow students to investigate geometric situations. Properties of triangles, quadrilaterals, and circles should receive particular attention. It is intended that students will use the traditional tools of compass and straightedge as well as dynamic geometry software that models these tools more efficiently and accurately, to assist in these investigations.
Geometry is meant to lead students to an understanding that reasoning and proof are fundamental aspects of mathematics and something that sets it apart from the other sciences.
Integrated Geometry

- Geometric Relationships
- Triangles
- Quadrilaterals
- Polygons
- Coordinate Geometry
- Transformational Geometry
- Circles
Integrated Geometry continued

- Informal and Formal Proofs
  - Euclidean
  - Logic
  - Coordinate Geometry
- Perimeter, Area, Volume, Surface area, lateral area
- Constructions
- Locus
Integrated Algebra 2
and
Trigonometry
*Algebra 2 and Trigonometry* is the capstone course of the three units of credit required for a Regents diploma. This course is a continuation and extension of the two courses that preceded it.

While developing the algebraic techniques that will be required of those students that continue their study of mathematics, this course is also intended to continue developing alternative solution strategies and algorithms.

Technology provides students the means to address a problem situation to which they might not otherwise have access.
Within this course, the number system will be extended to include imaginary and complex numbers.

The families of functions to be studied will include polynomial, absolute value, radical, trigonometric, exponential, and logarithmic functions.

Problem situations involving direct and indirect variation will be solved. Problems resulting in systems of equations will be solved graphically and algebraically.

Algebraic techniques will be developed to facilitate rewriting mathematical expressions into multiple equivalent forms.
Data analysis will be extended to include measures of dispersion and the analysis of regression that model functions studied throughout this course. Associated correlation coefficients will be determined, using technology tools and interpreted as a measure of strength of the relationship.

Arithmetic and geometric sequences will be expressed in multiple forms, and arithmetic and geometric series will be evaluated.

Binomial experiments will provide the basis for the study of probability theory and the normal probability distribution will be analyzed and used as an approximation for these binomial experiments.
Right triangle trigonometry will be expanded to include the investigation of circular functions. Problem situations requiring the use of trigonometric equations and identities will also be investigated.
Regents Exams

Format
Math A

Part I
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Part II
Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

Part III
Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

Part IV
Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [8]
Math B

Part I
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

Part II
Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

Part III
Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

Part IV
Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]
Integrated Algebra

The questions on the Regents Examination in Integrated Algebra will assess both the content and the process strands. Each question will be aligned to one content performance indicator but will also be aligned to one or more process performance indicators, as appropriate for the concepts embodied in the task. The examination will assess students’ conceptual understanding, procedural fluency, and problem-solving abilities rather than assessing knowledge of isolated skills and facts.

<table>
<thead>
<tr>
<th>Content Strand</th>
<th>% of Total Credits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Number Sense and Operations</td>
<td>6–10%</td>
</tr>
<tr>
<td>2) Algebra</td>
<td>50–55%</td>
</tr>
<tr>
<td>3) Geometry</td>
<td>14–19%</td>
</tr>
<tr>
<td>4) Measurement</td>
<td>3–8%</td>
</tr>
<tr>
<td>5) Probability and Statistics</td>
<td>14–19%</td>
</tr>
</tbody>
</table>
Schools must make a graphing calculator available for the exclusive use of each student while that student takes the Regents Examination in Integrated Algebra.
There will be 38 questions on the Regents Examination in Geometry. The table below shows the percentage of total credits that will be aligned with each content band.

<table>
<thead>
<tr>
<th>Content Strand</th>
<th>% of Total Credits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Relationships</td>
<td>8–12%</td>
</tr>
<tr>
<td>Constructions</td>
<td>3–7%</td>
</tr>
<tr>
<td>Locus</td>
<td>4–8%</td>
</tr>
<tr>
<td>Informal and Formal Proofs</td>
<td>41–47%</td>
</tr>
<tr>
<td>Transformational Geometry</td>
<td>8–13%</td>
</tr>
<tr>
<td>Coordinate Geometry</td>
<td>23–28%</td>
</tr>
<tr>
<td>Question Type</td>
<td>Number of Questions</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>Multiple choice</td>
<td>28</td>
</tr>
<tr>
<td>2-credit open ended</td>
<td>6</td>
</tr>
<tr>
<td>4-credit open ended</td>
<td>3</td>
</tr>
<tr>
<td>6-credit open ended</td>
<td>1</td>
</tr>
</tbody>
</table>

**Total points** 86

**Calculators**

Schools must make a graphing calculator available for the exclusive use of each student while that student takes the Regents Examination in Geometry.
There will be 39 questions on the Regents Examination in Algebra 2/Trigonometry.

<table>
<thead>
<tr>
<th>Content Strand</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1) Number Sense and Operations</td>
<td>6–10%</td>
</tr>
<tr>
<td>2) Algebra</td>
<td>70–75%</td>
</tr>
<tr>
<td>4) Measurement</td>
<td>2–5%</td>
</tr>
<tr>
<td>5) Probability and Statistics</td>
<td>13–17%</td>
</tr>
</tbody>
</table>
Question Types

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</tr>
</thead>
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<tr>
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<td>2-credit open ended</td>
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</tr>
<tr>
<td>4-credit open ended</td>
<td>3</td>
</tr>
<tr>
<td>6-credit open ended</td>
<td>1</td>
</tr>
</tbody>
</table>

Total points 88

Calculators

Schools must make a graphing calculator available for the exclusive use of each student while that student takes the Regents Examination in Algebra 2/Trigonometry.
Regents Examinations

Sample Questions
Part 2 #31
If a United States dollar is worth $1.41 in Canadian money, how much is $100 in Canadian money worth in United States money, to the nearest cent?

Part 4 #38
Angelo, Brandon, and Carl work in the same office. Angelo’s age is 4 years more than twice Carl’s age. Brandon is 5 years younger than Carl. The average of the three ages is 41. Find the age of each of the men.
[2] 70.92, and appropriate work is shown, such as a proportion. 
[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 70.92, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Angelo is 66, Brandon is 26, and Carl is 31, and appropriate work is shown, such as solving an equation or trial and error with at least three trials and appropriate checks.

3 Appropriate work is shown, but one computational error is made. Or 66, 26, and 31, and appropriate work is shown, but the solutions are not labeled or are labeled incorrectly.

2 Appropriate work is shown, but two or more computational errors are made.

2 Appropriate work is shown, but one conceptual error is made.

2 The trial-and-error method is used to find a correct solution, but only two trials and appropriate checks are shown. Or trial-and-error method is attempted and at least six systematic trials and appropriate checks are shown, but no solution is found.

Carl is 31, and appropriate work is shown, but the ages of the other men are not found.

2 An incorrect equation of equal difficulty is solved appropriately.

1 A correct equation is written, but no further correct work is shown.

0 No work is shown or the answers are not labeled or are labeled incorrectly. Or response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Part 2 #22
The accompanying table shows the enrollment of a preschool from 1980 through 2000. Write a linear regression equation to model the data in the table.

<table>
<thead>
<tr>
<th>Year (x)</th>
<th>Enrollment (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>14</td>
</tr>
<tr>
<td>1985</td>
<td>20</td>
</tr>
<tr>
<td>1990</td>
<td>22</td>
</tr>
<tr>
<td>1995</td>
<td>28</td>
</tr>
<tr>
<td>2000</td>
<td>37</td>
</tr>
</tbody>
</table>
Part 4 #33
Given: quadrilateral $ABCD$ with vertices $A(-2,2)$, $B(8,-4)$, $C(6,-10)$, and $D(-4,-4)$.
State the coordinates of $A'B'C'D'$, the image of quadrilateral $ABCD$ under a dilation of factor 2.
Prove that $A'B'C'D'$ is a parallelogram.
Part 2 #22

[2] \( y = 1.08x - 2125 \) or an equivalent equation is written.

[1] One conceptual error is made, such as writing a regression equation that is not linear.

[1] The expression \( 1.08x - 2125 \) is written, but no equation is written.

[1] The correct values are identified for \( a \) and \( b \), but no equation is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Part 4 #33

[6] The vertices $A'(-1,1), B'(4,-2), C'(3,-5),$ and $D'(-2,-2)$ are stated and a complete and correct proof that includes a conclusion is written.

[5] The vertices are stated, and a proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one reason is missing or is incorrect.

[5] A complete proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but the vertices of $A'B'C'D'$ are not stated.

[4] The vertices are stated, and a proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.
[3] The vertices are stated, and a proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two reasons are missing or are incorrect.

[2] The vertices are stated, and some correct relevant statements about the proof are made, but three or four statements or reasons are missing or are incorrect.

[1] The vertices \( A'(−1,1), \ B'(4,−2), \ C'(3,−5), \) and \( D'(−2,−2) \) are stated, but no proof is written.

[0] The “given” and/or the “prove” statements are rewritten in the style of a formal proof, but no further correct relevant statements are written. Or response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Part 2 # 35
A prom ticket at Smith High School is $120. Tom is going to save money for the ticket by walking his neighbor’s dog for $15 per week. If Tom already has saved $22, what is the minimum number of weeks Tom must walk the dog to earn enough to pay for the prom ticket?

39 Solve for x: \( \frac{x + 1}{x} = \frac{-7}{x - 12} \)
Thank You!